FOCUSING MIRRORS FOR GAUSSIAN BEAMS

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ABSTRACT

A new procedure is presented, that calculates the shape of focusing mirrors by controlling the phase distribution of Gaussian beams. The angle of incidence and three of the four beam parameters (beam waist radii and focal lengths of the incident and reflected beam) can be choosen arbitrarily.

1. INTRODUCTION

Mirrors rather than lenses can be used to compensate for the inherent growth of the diameter of Gaussian beams (figure 1). Compared to lenses mirrors give some advantages: Lenses cause losses due to the reflections at the interface between air and dielectric material, with mirrors this is not a problem. The losses caused by the reflection at the lossy metallic surface of a mirror are, in general small compared to the dielectric losses in the material of a lens. Since mirrors simultaneously focus and bend the beam, compact folded quasi optical systems can be constructed.

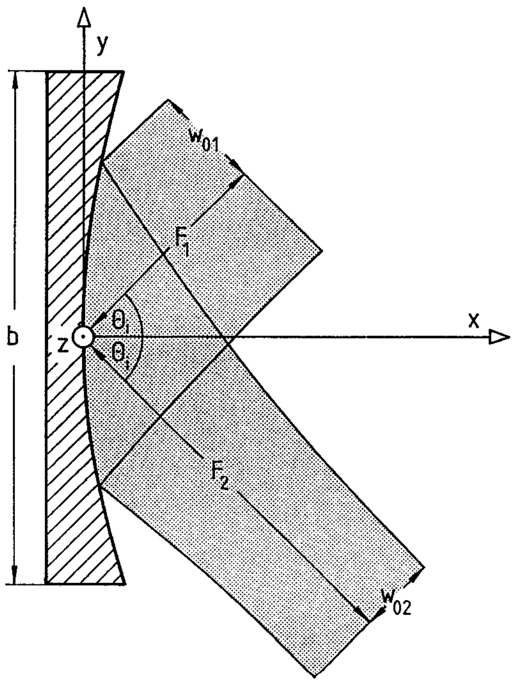

Fig. 1: The beam path of a focusing mirror

Focusing mirrors that are in common use have an ellipsoidal shape. The beam waists of the incident and the reflected beam are located on the straight lines from the center of the mirror to its two focal points. An ellipsoidal mirror, however, is only an approximation, that neglects the phase characteristic of the Gaussian beam.

This paper presents a new method for computing the optimal shape of focusing mirrors that are not subject to the above mentioned neglection.

2. DEFINITIONS

The term focusing mirror is used here to mean a loss-free metallic reflector that images an incident Gaussian beam with a beam waist radius w_{01}, into a reflected Gaussian beam with a beam waist radius w_{02}. The distances between the intersection points of the beam axes and the beam waists are denoted as focal lengths F_1 and F_2. The angle between the beam axes is $2 * \theta_i$ (figure 1).

The shape of the mirror depends on: the focal lengths (F_1, F_2), the beam waist radii (w_{01}, w_{02}) and the angle of incidence θ_i.

Considering only the fundamental Gaussian mode (TEM_{oo}), the following functions are well known from Gaussian beam theory (1).

The radii of the spherical phase fronts are given by (figure 2):

$$R(w_0, \lambda, \zeta) = \zeta \cdot \left[1 + \left(\frac{\pi w_0^2}{\lambda \zeta} \right)^2 \right] \qquad (1)$$

The beam radii are given by :

$$w(w_0,\lambda,\zeta) = w_0 \cdot \left[1 + \left(\frac{\lambda\,\zeta}{\pi\,w_0^2}\right)^2\right] \quad (2).$$

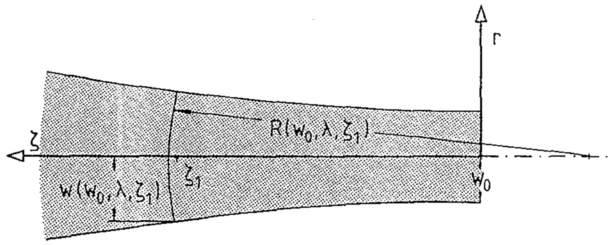

Fig. 2: Beam parameter of a Gaussian beam

The amplitudes of the electric or magnetic field strengths are proportional to the function A :

$$A(w_0,\lambda,\zeta,r) = \frac{w_0}{w(w_0,\lambda,\zeta)} \cdot \exp\left[-\frac{r^2}{w^2(w_0,\lambda,\zeta)}\right] \quad (3).$$

The phase shift referred to the beam waist is given by :

$$\varphi(w_0,\lambda,\zeta,r) = \quad (4),$$

$$-\frac{2\pi}{\lambda}\cdot\zeta - \frac{\pi\cdot r}{\lambda\cdot R(w_0,\lambda,\zeta)} + \arctan\left(\frac{\lambda\cdot\zeta}{\pi\cdot w_0^2}\right)$$

where λ denotes the free space wavelength.

3. PHASE CONDITION AND AMPLITUDE CONDITION

To function properly, the mirror must reflect the incident beam in a way that the plane phase front at the beam waist of the incident beam is imaged into a plane phase front at the beam waist of the reflected beam. Referring to the phase shift along the beam axes (figure 3, $r_1 = 0$, $r_2 = 0$) the phase condition is:

$$\varphi[w_{01},\lambda,\zeta_1(\bar{C}),r_1(\bar{C})] + \varphi[w_{02},\lambda,\zeta_2(\bar{C}),r_2(\bar{C})] =$$

$$\varphi(w_{01},\lambda,F_1,0) + \varphi(w_{02},\lambda,F_2,0) \quad (5).$$

At the loss-free mirror surface, the amplitudes of the field strengths of the incident beam must equal the amplitudes of the field strengths of the reflected beam. Therefore, the amplitude condition is:

$$A[w_{01},\lambda,\zeta_1(\bar{C}),r_1(\bar{C})] = \quad (6)$$

$$A[w_{02},\lambda,\zeta_2(\bar{C}),r_2(\bar{C})] \cdot U_2$$

In this expression, U_2 is a constant of proportionality, that is considered later.

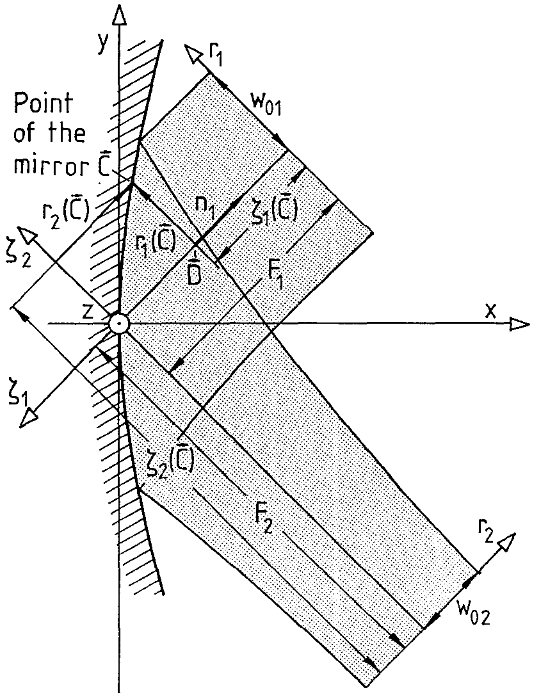

Fig. 3: Calculating the phase and amplitude condition for a focusing mirror

Both the phase condition equation (5) and the amplitude condition equation (6) define mirror surfaces which, unfortunately, are different in general. If the amplitude condition is satisfied, the Huyghens sources, which generate the reflected beam, have the proper amplitude but a incorrect phase distribution. If, on the other hand, the phase condition is satisfied, the Huyghens sources have an incorrect amplitude distribution but a proper phase distribution. Hence, in general, it is impossible for a reflector to transform all of the incident energy into the desired Gaussian mode for the reflected energy.

Gaussian beams have a relatively smooth change of their amplitude. Therefore phase optimized mirrors normally causes relatively small amplitude errors, that are tolerable in practice provided that the beam radii of the incident and reflected beam are matched to each other. The method used here for matching is to make the beam radii equal at the center of the mirror. This means:

$$w_1 = w(w_{01},\lambda,F_1) = w_2 = w(w_{02},\lambda,F_2) \quad (7)$$

If w_{01} , w_{02} , F_1 , λ are given, equation (7) results in:

$$F_2 = w_{02}\cdot\left[\frac{\pi^2}{\lambda^2}\cdot(w_{01}^2 - w_{02}^2) + \frac{F_1^2}{w_{01}^2}\right]^{\frac{1}{2}} \quad (8)$$

If w_{01} , F_1 , F_2 , λ are given, equation (7) results in:

$$w_{02} = \left\{ H \pm \left[H^2 - \left(\frac{\lambda F_2}{\pi}\right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$

with: (9).

$$H = \frac{(w_{01}^2 \cdot \pi)^2 + (\lambda F_1)^2}{2(w_{01} \cdot \pi)^2}$$

A negative radicant in equations (8) or (9) indicates that with the given beam parameters it is impossible to match the amplitudes.

The proportionality constant from equation (6) can be calculated from the requirement that at the center of the mirror the incident beam and the reflected beam must have equal amplitudes of their field strengths. Therefore U_2 yields:

$$U_2 = \frac{w_{01} \cdot w(w_{02}, \lambda, F_2)}{w_{02} \cdot w(w_{01}, \lambda, F_1)} \qquad (10).$$

If the choosen beam parameters satisfy equation (7) exactly, equation (10) simplifies to:

$$U_2 = \frac{w_{01}}{w_{02}} \qquad (11).$$

On the other hand amplitude optimized mirrors often cause very high phase errors that easily exceed 180 degrees so that these mirrors are not satisfactory (2).

4. CALCULATING THE SHAPE OF THE MIRROR

Fabrication of geometrically complex mirrors is performed on a numerically controlled milling machine. Therefore it is appropriate to calculate the shape of the mirror point by point. This means that for a given pair of coordinates (C_y , C_z) the third coordinate C_x has to be calculated. To do this the following procedure may be used.

a) Match the beam radii at the center of the mirror. Equation (8) is used to calculate the focal length F_2 for given beam waist radii w_{01} , w_{02} and focal length F_1 or equation (9) is used to calculate the beam waist radius w_{02} for given beam waist radius w_{01} and focal lengths F_1 , F_2 .

b) Calculate the proportionality constant U_2 using equations (10) or (11).

c) Select the pairs of coordinates (C_y , C_z). The mirror should be large enough that a suffcent portion of the incident energy falls upon the mirror. Therefore the height h of the mirror (z - dimension) is choosen to be:

$$h = S \cdot w(w_{01}, \lambda, F_1) = S \cdot w(w_{02}, \lambda, F_2) \qquad (12)$$

and the width b of the mirror (y - dimension, figure 1) is choosen to be:

$$b = \frac{h}{\cos \theta_i} \qquad (13).$$

The factor S determines the amount of energy that falls upon the mirror. Usual values for S are between 3 and 4. I.e if S = 3 more than 98.89 % of the incident energy falls upon the mirror.

d) A computer program is used to calculate the coordinate C_x for every pair of coordinates (C_y , C_z) such that equation (5) is satisfied.

The coordinates of the incident beam $r_1(\bar{C})$ and $\zeta_1(\bar{C})$ that are associated with the point $\bar{C} = (C_x, C_y, C_z)$ of the mirror can be calculated as follows (figure 3):

Point $\bar{D}$ in figure 3 is the point of intersection between the plane:

$$\bar{n}_1 \cdot \bar{r} - \bar{n}_1 \cdot \bar{C} = 0 \qquad (14)$$

and the straight line:

$$\bar{r} = \nu \cdot \bar{n}_1 \qquad (15).$$

The result is:

$$\bar{D} = (C_x \cdot \cos \theta_i + C_y \cdot \sin \theta_i) \cdot \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \\ 0 \end{bmatrix} \qquad (16).$$

Knowing $\bar{D}$ the coordinates can be calculated with:

$$\zeta_1(\bar{C}) = F_1 - \frac{D_x}{\cos \theta_i} = F - \frac{D_y}{\sin \theta_i} \qquad (17)$$

$$r_1(\bar{C}) = |\bar{D} - \bar{C}| \qquad (18).$$

The coordinates $r_2(\bar{C})$ and $\zeta_2(\bar{C})$ of the reflected beam can also be calculated using equations (16), (17) and (18) provided that the following substitutions are made:

$$r_1 \rightarrow r_2 \quad , \quad \zeta_1 \rightarrow \zeta_2 \quad , \quad \theta_i \rightarrow -\theta_i$$

e) Check the amplitude error. Since the mirror surface is calculated to satisfy the phase condition equation (5) the amplitude error should be checked with equation (6). Normally the amplitude error grows with increasing distance from the center of the mirror. On the other hand the amplitude of the beam decreases rapidly with increasing distance from the center of the mirror, so that the amplitude error remains tolerable in practice.

It is evident that these mirrors are symmetrical with regard to the xy-plane. If in addition:

$$w_{01} = w_{02} \quad \text{and} \quad F_1 = F_2 \tag{19}$$

then the mirrors are symmetrical with regard to the xz-plane also. If $\Theta_i = 0$ then the mirrors are rotationally symmetrical with regard to the x-axis.

5. CONCLUSION

The method presented in this paper is able to compute the shape of focusing mirrors that image an incident Gaussian beam into a reflected Gaussian beam. The angle of incidence and three of the four quantities w_{01}, F_1, w_{02}, F_2 (beam waist radii and focal lengths of the incident and the reflected beam respectively) can be choosen arbitrarily. The remaining forth quantity is computed so that the beam radii of both beams are matched to each other at the center of the mirror.

Although the method was developped for the fundamental Gaussian mode it would work for higher-order modes too if equations (3) and (4) are replaced by appropriate equations i.e. for TEM_{ml} modes described in cylindrical coordinates (r, ψ, ζ) equation (3) would read:

$$A(w_0,\lambda,\zeta,r,\psi) = \frac{w_0}{w}\cdot\left(\sqrt{2}\,\frac{r}{w}\right)^l\cdot L_m^{(l)}\left(2\frac{r^2}{w^2}\right)\cdot \\ \cdot\exp\left(-\frac{r^2}{w^2}\right)\cdot\cos(l\cdot\psi-\psi_0) \tag{20}$$

where $L_m^{(l)}$ denote the generalized Laguerre polynomials (i.e. (3)) and equation (4) would read:

$$\varphi(w_0,\lambda,\zeta,r) = -\frac{2\pi}{\lambda}\cdot\zeta - \frac{\pi r^2}{\lambda R} + \\ + (2m+l+1)\cdot\arctan\left(\frac{\lambda\cdot\zeta}{\pi w_0^2}\right) \tag{21}.$$

Since only the fundamental Gaussian mode is in common use in quasioptical mm-wave techniques, mirrors for higher order modes have not been examined by the author.

6. REFERENCES

(1) Kogelnik, H.; Li, T.: Laser beams and resonators. Proceedings of the IEEE vol. 54 (1966) No. 10, S. 1312 - 1329.

(2) Boheim, M.: A remark on focusing mirrors. 3. Workshop of the MTT-Chapter of the West Germany Section of the IEEE, proceedings of the 1st European Quasi-optical conference May 12-13, 1987.

(3) Abramowitz, Milton; Stegun, Irene A.: Handbook of mathematical functions, 9. edition. New York: Dover Publications Inc., 1970.